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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

THIRD SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018

Course Code: MA201

Course Name: LINEAR ALGEBRA AND COMPLEX ANALYSIS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer any two full questions, each carries 15 marks Marks

1 a) Let $f(z) = u(x, y) + iv(x, y)$ be defined and continuous in some neighbourhood of a point $z = x + iy$ and differentiable at z itself. Then prove that the first order partial derivatives of u and v exist and satisfy the Cauchy – Riemann equations. (7)

b) Prove that $u = \sin x \cosh y$ is harmonic. Hence find its harmonic conjugate. (8)

2 a) Find the image of the region $\left|z - \frac{1}{3}\right| \leq \frac{1}{3}$ under the transformation $w = \frac{1}{z}$ (8)

b) Find a linear fractional transformation which maps $-1, 0, 1$ onto $1, 1 + i, 1 + 2i$. (7)

3 a) Check whether the function $f(z) = \begin{cases} \frac{Re\,(z^2)}{|z|^2} & if\ z \neq 0 \\ 0 & if\ z = 0 \end{cases}$ is continuous at $z = 0$. (7)

b) Find the image of the x-axis under the linear fractional transformation $w = \frac{z+1}{2z+4}$ (8)

PART B

Answer any two full questions, each carries 15 marks

4 a) Evaluate $\int_C Im(z^2)dz$ where C is the triangle with vertices $0, 1, i$ counter-clockwise. (7)

b) Using Cauchy's Integral Formula, evaluate $\int_c \frac{z^2}{z^3-z^2-z+1}\,dz$ where c is taken counter-clockwise around the circle: (8)

i) $|z + 1| = \frac{3}{2}$ ii) $|z - 1 - i| = \frac{\pi}{2}$

5 a) Determine and classify the singular points for the following functions: (7)

i) $f(z) = \frac{\sin z}{(z-\pi)^2}$ ii) $g(z) = (z + i)^2 e^{\left(\frac{1}{z+i}\right)}$

b) Evaluate $\int_{-\infty}^{\infty} \frac{1}{(1+x^2)^3}\,dx$. (8)

6 a) Evaluate $\int_C \frac{\tan z}{z^2-1}\,dz$ counter clockwise around c: $|z| = \frac{3}{2}$ using Cauchy's Residue Theorem. (7)

b) Find all Taylor series and Laurent series of $f(z) = \frac{-2z+3}{z^2-3z+2}$ with centre 0 in (8)

i) $|z| < 1$ ii) $1 < |z| < 2$.

PART C

Answer any two full questions, each carries 20 marks

7 a) Solve the system of equations by Gauss Elimination Method: (8)

$3x + 3y + 2z = 1, \quad x + 2y = 4, \; 10y + 3z = -2, \quad 2x - 3y - z = 5.$

b) Prove that the vectors $(1, 1, 2), (1, 2, 5), (5, 3, 4)$ are linearly dependent. (6)

c) Prove that the set of vectors $V = \{(v_1, v_2, v_3) \in \mathbb{R}^3 : -v_1 + v_2 + 4v_3 = 0\}$ a vector space over the field $\mathbb{R}$. Also find the dimension and the basis. (6)

8 a) Find the Eigen values and the corresponding Eigen vectors of $A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ (8)

b) What kind of conic section is given by the quadratic form $7{x_1}^2 + 6x_1x_2 + 7{x_2}^2 = 200$. Also find its equation. (6)

c) Determine whether the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\theta \\ 0 & sin\theta & cos\theta \end{bmatrix}$ symmetric, skew-symmetric or orthogonal. (6)

9 a) Reduce the matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ to Row Echelon Form and hence find its rank. (8)

b) Diagonalize $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ (12)

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